関数電卓による波力簡易計算法

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Simplified Method to Calculate Wave Force Using a Scientific Electronic Calculator

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In coastal fisheries of Japan, facilities such as set-nets, net cages, and artificial reefs are commonly used. Designing of such facilities requires calculation of the wave force. However, wave force calculations have previously been made using a computer because the mathematical treatment of waves is complex and even in the easiest wave theory, a hyperbolic function was applied. Furthermore it needed an integral calculation incorporating the water depth. On the other hand, currently available scientific electronic calculators are commonly equipped with a hyperbolic function and memory function. Consequently, we devised a simplified method of wave force calculation using a scientific electronic calculator and compared it with the conventional method using a computer. We regarded such fishery facilities as submerged cylinders for the sake of simplicity and calculated the wave force acting on them by both methods. It became clear that we can regard that there is little difference between wave force estimated by the simplified method and the one by the conventional method if the cylinder length is smaller than 0.167 times the wave length. That means the simplified method is effective. Finally, the calculation by the simplified method requires only a scientific electronic calculator, which allows the wave calculation to be easily done by hand. Furthermore the simplified method is applicable for use in lectures about wave force and thus has applications in education.

Key words : wave forces, wave particle velocity, cylindrical bodies, submerged cages.

1 Introduction

Development of coastal fisheries is one of the important tasks for the Japanese fishing industry due to the setting of 200 nautical miles fishery zone[1]. In coastal fisheries of Japan, facilities such as set-nets, net cages, and artificial reefs are commonly used. The wave force on these facilities in coastal fisheries is also an important problem, although it was of little concern in the offshore and deep sea fishery, which mainly uses fishing boats.

Designing of such facilities therefore requires calculation of the wave force. However, wave force calculations have previously been made using a computer because the mathematical treatment of waves is complex and even in the easiest wave theory, a hyperbolic function was applied. Furthermore it needed an integral calculation incorporating the water depth[2].

Present scientific electronic calculators are commonly equipped with a hyperbolic function and memory function. Consequently, we devised a simplified method of wave force calculation using a scientific electronic calculator. Comparing it with the conventional method using a computer, we confirmed it is appropriate for use.

2 Calculation of the wave force on a submerged body using small amplitude wave theory

(1) Elements of waves

As wave force differs with the type and features of structures, it is classified in many kinds of theories[3]. However we applied the small amplitude wave theory because of the wave calculation on the fishery facilities. Elements of waves are shown in Fig. 1 . In the figure, T :

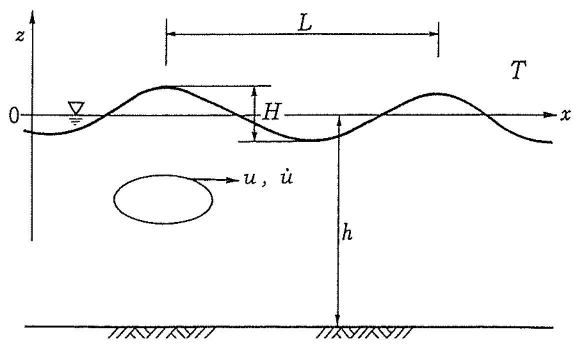

Fig. 1 . Elements of waves.

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wave period, L : wave length, H : wave height, h : water depth, z : depth from the water surface (vertically upside indicates positive), u : horizontal velocity of water particles of waves, $\dot{u}$: horizontal acceleration of water particles of waves.

In the small amplitude wave theory, elements of waves used for wave calculation are expressed by the following equations : (1) to (6), where g is the acceleration of gravity [4]. In that theory, waves are classified into deep water waves and shallow water waves. In order to distinguish deep water waves from shallow water waves, the subscript 0 is attached to each symbol representing deep water waves. Furthermore, maximum values are applied to the velocity and acceleration of water particles because maximum values of wave are only needed in the wave force calculation as described later.

・wave length

$$\begin{cases} \text{(Deep)} \quad L_0 = \dfrac{gT^2}{2\pi} & (1) \\ \text{(Shallow)} \quad L = \dfrac{gT^2}{2\pi} \cdot \tanh \dfrac{2\pi}{L} h & (2) \end{cases}$$

・maximum horizontal velocity of water particles

$$\begin{cases} \text{(Deep)} \quad u_{0\max} = \dfrac{\pi H_0}{T} \cdot e^{(2\pi z/L_0)} & (3) \\ \text{(Shallow)} \quad u_{\max} = \dfrac{\pi H}{T} \cdot \dfrac{\cosh(2\pi(h+z)/L)}{\sinh(2\pi h/L)} & (4) \end{cases}$$

・maximum horizontal acceleration of water particles

$$\begin{cases} \text{(Deep)} \quad \dot{u}_{0\max} = \dfrac{2\pi^2 H_0}{T^2} \cdot e^{(2\pi z/L_0)} & (5) \\ \text{(Shallow)} \quad \dot{u}_{\max} = \dfrac{2\pi^2 H}{T^2} \cdot \dfrac{\cosh(2\pi(h+z)/L)}{\sinh(2\pi h/L)} & (6) \end{cases}$$

As the above equations show, a hyperbolic function can be applied to calculate the elements of waves. Therefore in the past, a computer was indispensable to calculate the elements of waves. However at present, using a scientific electronic calculator equipped with the hyperbolic function, it is very easy to calculate the elements of waves.

(2) Calculation of wave length of shallow water waves

The wave length of shallow water waves (L) is necessary to calculate the horizontal velocity and acceleration of water particles of waves. However, as shown in equation (2), L is needed to calculate L. This means that the direct calculation of L is impossible. On that point, in the past, a computer was also indispensable to calculate L[1]. But at present, using a scientific electronic calculator, we are able to calculate L very easily by iterative calculations. In addition, if we use the memory function of a scientific electronic calculator, the calculation is far easier. We will show the method as follows.

First, it is necessary to decide which equation, for deep water waves or shallow water waves should be applied under the given conditions. The criterion of judgment is whether the relative water depth (h/L) is larger than 0.5. That is, $h/L < 0.5$ indicates the application of shallow water waves. We can use L_0 as L for this discrimination. The L_0 can be easily calculated by equation (1) if the wave period (T) is given.

Secondly, we apply the L_0 to L of the right side member of equation (2) as expedient. Let the calculated L be L_1. Additionally, applying this L_1 to L of the right side member of equation (2) again and get L_2. Repeatedly, using this L_2, we can get L_3. In this way, using the iterative calculations we get L_i from L_{i-1}.

Then the value of L_i changes fluctuatingly and converges to a certain value (Fig. 2). If the difference between L_i

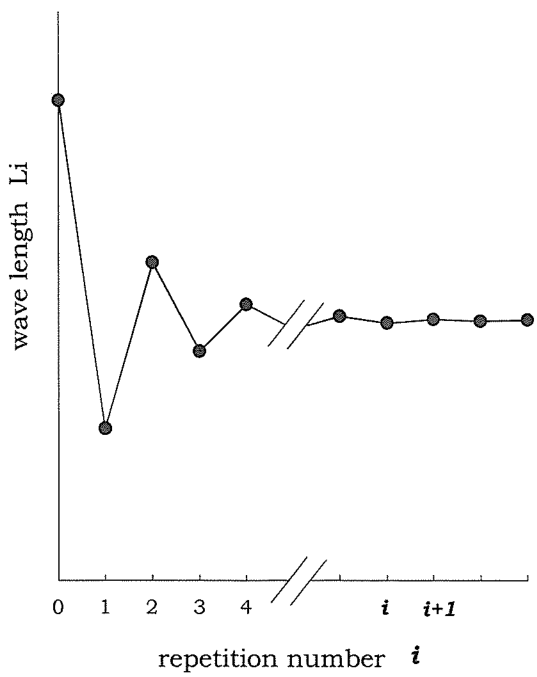

Fig. 2 . Relationship between repetition number (i) and wave length for shallow water (L_i).

and L_{i-1} becomes smaller than the allowable error, we regard L_i as the wave length L of shallow water waves. The number of iterative calculations will be smaller than 5 or 6 unless the relative water depth is very small. If we input the values of L_0 $(=gT^2/2\pi)$ and $2\pi h$ into the memory of the scientific electronic calculator, and use them to calculate each L_i, we are able to calculate very efficiently. Furthermore, if we input the calculated values of L_i into the memory and use them to calculate the next L_{i+1}, the calculation can be achieved very rapidly.

(3) Calculation of wave force

Wave force in the horizontal direction acting on a submerged body in progressive waves can generally be expressed by equation (7) [5]. Here we consider about the wave force on a cylinder set perpendicularly in the sea (Fig. 3).

$$dF = \frac{1}{2}\cdot C_D\cdot \rho \cdot u\cdot |u|\cdot dS + C_M\cdot \rho \cdot \dot{u}\cdot dV \qquad (7)$$

where : dF (horizontal wave force acting on the section with vertical distance dz), C_D (drag coefficient), C_M (mass coefficient), dS (projected cross sectional area of dz section of the body to the direction of wave flow), dV (volume of dz section of the body), u (horizontal velocity of water particles of waves), $\dot{u}$ (horizontal acceleration of water particles of waves).

This relation was suggested by Morison et al. Wave force is expressed as the sum of the drag force from water particle velocity and the mass force from water particle acceleration.

The requisite wave force in designing the facility is the maximum wave force. Then the maximum drag force ($dF_{D\max}$) and the maximum mass force ($dF_{M\max}$) acting on dz are expressed by equation (8) and (9) respectively.

$$dF_{D\max} = \frac{1}{2}\cdot C_D\cdot \rho \cdot u_{\max}{}^2 \cdot dS \qquad (8)$$

$$dF_{M\max} = C_M\cdot \rho \cdot \dot{u}_{\max}\cdot dV \qquad (9)$$

Namely,

$$F_{D\max} = \int_{z=0}^{z=-h} dF_{D\max} \qquad (8)'$$

$$F_{M\max} = \int_{z=0}^{z=-h} dF_{M\max} \qquad (9)'$$

As the velocity of the water particles reaches 90° out of phase of its acceleration, drag force F_D reaches the maximum ($F_{D\max}$) and mass force F_M becomes 0 at the peak of waves. However at the still water level of wave shape, mass force F_M is at the maximum ($F_{M\max}$) and the drag force F_D becomes 0. $u_{\max}$ and $\dot{u}_{\max}$ in the formulas (8) and (9) are given by the equation (4) and (6).

From the composition of single vibrations $F = F_{D\max}\cdot\sin\theta + F_{M\max}\cdot\cos\theta$. Therefore, the maximum wave force ($F_{\max}$) can be calculated by equation (10).

$$F_{\max} = \sqrt{(F_{D\max})^2 + (F_{M\max})^2} \qquad (10)$$

(4) Simplified method of the wave force calculation

Since the velocity and acceleration of water particles of waves are functions of the water depth from the water surface (z) shown by equation (3) to (6), calculation of the wave force acting on the submerged body is calculated by integration from the depth of water of the upper part of the body (z_1) to the lower part of the body (z_2). Therefore the calculation must be done using a computer. But here, as a simplified method, wave force is calculated by using the water particle velocity and acceleration of waves at the central part of the body (z_C) as follows.

① Distinguish whether shallow water waves or deep water waves

$$h/L_0 \quad \geqq \text{or} < \quad 1/2$$

② Calculation of water particle velocity and accelera-

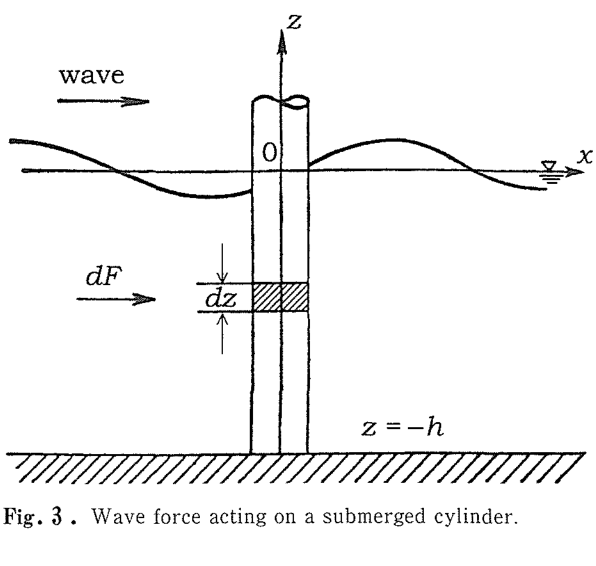

Fig. 3. Wave force acting on a submerged cylinder.

tion of waves

(a) Deep water waves

$$\begin{bmatrix} L_0 \\ (1) \end{bmatrix} \longrightarrow \begin{bmatrix} u_{0\max} & (3) \\ \dot{u}_{0\max} & (5) \end{bmatrix}$$

(b) Shallow water waves

$$\begin{bmatrix} L \\ (2) \end{bmatrix} \longrightarrow \begin{bmatrix} u_{\max} & (4) \\ \dot{u}_{\max} & (6) \end{bmatrix}$$

③ Calculation of the maximum values of drag force, mass force and wave force

$$\begin{bmatrix} F_{D\max} & (8) & (dS \rightarrow S) \\ F_{M\max} & (9) & (dV \rightarrow V) \end{bmatrix} \longrightarrow \begin{bmatrix} F_{\max} \\ (10) \end{bmatrix}$$

※In the calculation of water particle velocity and acceleration of waves, z_c is used as z. Instead of dS and dV, projected cross sectional area of the whole body (S) and volume of the whole body (V) are used.

3 Comparison of the calculation by integration with the calculation by simplified method of the wave force

When we regard fishery facilities such as net cages and floating fish aggregating devices as submerged bodies and calculate the wave force acting on them, let us consider those facilities as cylinders for the sake of simplicity. Namely we calculate the wave force acting on a cylinder (diameter: D, length: ℓ), which is set in water (water depth of the sea bottom: h, water depth from the water surface at the central part of the cylinder: z_c) sustaining wave (wave period: T, wave height: H).

We set the wave conditions as follows considering the observed values around Japan[6].

$T =$ 6, 9, 12, 15 sec.

$H =$ 4, 8, 12 m

$h =$ 20, 40, 60 m

We also set the cylinder conditions as follows

$D =$ 2, 6, 10 m

$\ell =$ 4, 8, 12, 16, 20 m

$z_c = \ell/2 \sim h/2$ m

Although values in an unsteady flow originally must be applied to C_D in equation (8), from the viewpoint of practical use, values in a steady flow are generally applied[5]. C_D of cylinders in finite length in a steady flow changes according to the slenderness (ℓ/D) of cylinders[7]. Under the condition above-mentioned, the slenderness (ℓ/D) changes from 0.1 to 2.5. Therefore, in this paper, we used C_D of cylinders according to slenderness (ℓ/D) for the calculation of wave force. We used 2.0 for C_M of cylinders.

Under these conditions, we carried out the calculation of wave force by the simplified method and by integration from the depth of water of the upper part of the cylinder (z_1) to the lower part of the cylinder (z_2) using a personal computer. We applied Simpson formula to the integral calculation[8].

We made the calculated value by the simplified method F_s and one by integration F_i respectively. In order to compare the difference between the two values, we examined the relationship between F_i/F_s and other factors which were made dimensionless. We can get 15 dimensionless factors as follows, H/L, h/L, D/L, ℓ/L, z_c/L, h/H, D/H, ℓ/H, z_c/H, D/h, ℓ/h, z_c/h, ℓ/D, z_c/D, z_c/ℓ.

As the result, it was clarified that F_i/F_s is closely related to ℓ/L and is more remotely related to other 14 factors. We show the relationship between ℓ/L and F_i/F_s in Fig. 4. In Fig. 4, F_i/F_s increases in proportion to the square of ℓ/L. Although there is some variation of F_i/F_s for the same value of ℓ/L, it is due to the influence of other factors. Then we examined the relationship between F_i/F_s and other factors for the same ℓ/L. However we could not determine any other relationship. Although the range of variation is large when ℓ/L is large, it converges to zero as ℓ/L becomes small.

As T is a function of L (equation (1) and (2)), naturally F_i/F_s at small T (= 6 seconds) are totally distributed on the right side and F_i/F_s at large T (= 15 seconds) are distributed on the left side. But they completely overlap with each other. A solid line of the two curves in the figure indicates a regression curve of the total data (number is 2,304) and the correlativity is very high ($\rho = 0.995$). On the other hand, the broken line indicates the line which connects upper limit of data. Former is shown by a regres-

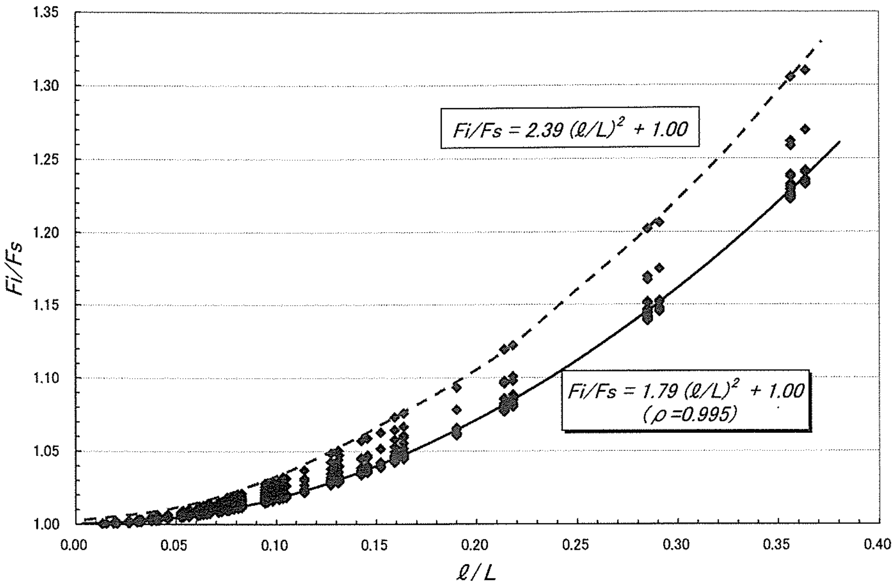

Fig. 4 . Relationship between ℓ/L and F_i/F_s. F_i : the calculated value by integration, F_s : the calculated value by the simplified method. The solid line indicates a regression curve of the total data (number is 2,304). In the regression equation, F_i/F_s becomes 1.05 in the case of ℓ/L equals to 0.167.

sion equation ① and latter is shown by a regression equation ②.

$$F_i/F_s = 1.79 \cdot (\ell/L)^2 + 1.00 \qquad ①$$

$$F_i/F_s = 2.39 \cdot (\ell/L)^2 + 1.00 \qquad ②$$

We consider that there is no difference between the two values, value by the simplified method F_s and the one by integration F_i, when the difference is smaller than 5 %, that is F_i/F_s is smaller than 1.05. In the regression equation ①, F_i/F_s becomes 1.05 in the case of ℓ/L equals to 0.167. That is, if the cylinder length (ℓ) is smaller than $0.167 \cdot L$, we can regard that there is no difference between the two values. When we put $\ell/L = 0.167$ into the regression equation ②, we can get $F_i/F_s = 1.066$. This means that the upper limit of F_i/F_s at $\ell/L = 0.167$ becomes 1.066. We can also regard there might be no difference between the two values.

Data in the case of $\ell/L > 0.167$ account for 13.7 percent of all data. However, on close inspection by period, the case of $T = 6$ seconds is main. Data in the case of $T = 6$ account for 53.1 percent and they account for 13.3 percent of all data. On the other hand, data of $\ell/L > 0.167$ in the case of $T = 9$ account for 0.4 percent of all data. Therefore in the wave, whose period is larger than 9 seconds, almost F_i/F_s become smaller than 1.05 under the condition above-mentioned. Namely we can regard that there is no difference between F_s and F_i under such conditions. Consequently it is effective that the simplified method about wave calculation which we proposed in this paper.

Furthermore, spheres are also often regarded representing submerged bodies. If we compare the calculated wave force on spheres by the simplified method against the one by integration, it is clear that there is far less difference between the two methods than in the case of cylinders, because the projected cross sectional area and volume of dz, which is the section at water depth at the central part of the body, become maximum in the case of spheres. That means that the simplified method about wave force calculation on a sphere is more effective.

4 Conclusion

We took cylinders representing fishery facilities and compared the calculated wave force on them by integration using a computer against one by the simplified method using a scientific electronic calculator, which we proposed in this paper. The latter is the calculation using water particle velocity and acceleration of waves at water depth from the water surface at the central part of the cylinder. It became clear that we can regard that there is no difference between the wave force by integration and the one by simplified method if the cylinder length is smaller than 0.167 times of wave length under the condition above-mentioned. This means that the simplified method about wave calculation which we propose in this paper is effective. Thus, the calculation by the simplified method needs only a scientific electronic calculator, and it can be easily done by hand. It might be very convenient for designing fishery facilities against waves. Furthermore it is applicable for use in lectures about wave force and thus has applications in education.

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関数電卓による波力簡易計算法

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定置網・増養殖施設・魚礁などの各種水産施設の設計には波力計算が伴うが，本来波力計算においては，コンピュータによる積分計算が避けられないのが実情であった。一方，現在の関数電卓には双曲線関数機能や複数のメモリー機能を付加したものが一般的である。そこで筆者らは，関数電卓を使った波力の簡易計算法を考え，従来のコンピュータを用いての積分計算法と比較した。水産施設を水中に鉛直に設置された円柱と考え，それらに作用する波力を二つの方法で算出した。その結果，円柱の長さが波長の0.167倍より小さければ，簡易計算値と積分計算値とに差がないことが示された。すなわち関数電卓を用いた簡易計算法での波力計算が有効であることが明らかになった。